Medieval Weapons of War in the Physics Curriculum

Kevin M. Crosby, Dan Lyons, Carrie Pinter, and Ann Randolph
Departments of Physics, Carthage College, Kenosha, WI 53158
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Students in the introductory mechanics course at Carthage College designed and built a trebuchet, which is a medieval counterweight catapult whose history is shrouded in mystery. While there are a few surviving drawings of the device, including those of Leonardo da Vinci, its design details are largely lost. Our recreations of the trebuchet were preceded by a Newtonian analysis of three variations of the trebuchet. Students in our mechanics course apply conservation principles to the trebuchet models, and make predictions regarding the effective range of each of the three models. Experiments and computer simulations corroborate the calculations.

I. INTRODUCTION

In 1304 Edward I mounted a daring attack against the Castle of Stirling in Scotland for which he ordered the construction of a devastating new weapon. The siege against Castle Stirling was successful, but there are few surviving records of the design of the weapon used in the siege. Historians believe it to have been a version of what later appears in Leonardo da Vincis drawings as a trebuchet. The trebuchet is a medieval siege weapon resembling a catapult, and used to launch massive projectiles at Castle walls from hundreds of yards or more. However, the surviving sketches of these weapons show devices with impractical sizes and proportions. Was such a weapon ever built? Was the trebuchet the decisive factor in the Stirling siege? Students in our introductory and advanced mechanics courses address the scientific aspects of these questions by constructing and analyzing a scale model trebuchet. This work grew out of a project in our case-studies based introductory laboratory curriculum.[1]

Essentially a giant seesaw with a massive counterweight on one end and a sling carrying the projectile on the other end, the trebuchet is a relatively simple device to build in model form, but can convincingly demonstrate application of conservation laws and rotational dynamics in dramatic new ways. A careful analysis of the trebuchets dynamics reveals many interesting features likely discovered by the original engineers through trial and error long before Newton. Some of these intriguing design issues will be discussed in the analysis section below.

Naturally, the prospect of building a weapon of mass destruction (by 14th Century standards, at least) is appealing to many students, some of whom are pictured in Fig. 1 with two of the instruments they designed and built. Inspired by a PBS broadcast[2] on modern recreations of trebuchets and a related article in Smithsonian magazine,[3] the three student authors pictured in Fig. 1 set out to construct and analyze a series of model trebuchets. We have since used the trebuchet in both the introductory courses in mechanics as well as in the advanced mechanics course as a novel context in which to address conservation laws, Lagrangian methods, and the pre-history of physics.

FIG. 1: The three student authors (from left to right: C. P., D. L., and A. R.) are shown with two of the scale model trebuchets they built for the project.

Three variations on a theme are illustrated in Fig. 2. Figure 2(a) shows an illustration of a treb with a fixed counterweight mounted onto the end of the throwing arm, while Fig. 2(b) shows a treb with a swinging counterweight affixed to the throwing arm by a pin. Finally, a treb with a rolling base is shown in 2(c). Given the same dimensions, counterweight and projectile masses, which design is more efficient in converting the stored gravitational potential energy into projectile kinetic energy? How can elementary physics be used to improve the design of the trebuchet? We set for ourselves the task of answering these questions through experiment, simulation, and through application of Newtonian dynamics.

II. CONSTRUCTION OF A MODEL TREBUCHET

A single treb was built from the template shown in Fig. 2(a). The base was constructed of 1/4 plywood. The footprint of the treb is a rectangle 40 cm × 22 cm, while the sides are plywood triangles 60 cm tall. A cylindrical wooden dowel 55 cm long and 3 cm in diameter was used for the throw arm. A 15 cm long lag bolt was fitted to one end to hold the fixed counterweight. For the

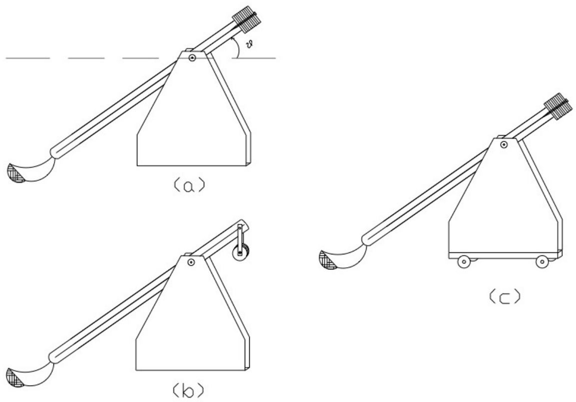

FIG. 2: Three variations on the trebuchet design are: a fixed counterweight, fixed base trebuchet (a), a swinging counterweight, fixed base trebuchet (b), and a fixed counterweight, rolling base trebuchet (c).

swinging counterweight configuration, the lag bolt was removed and a 2 bolt driven through the diameter of the arm at one end was fitted with bearings and carried the carriage for the counterweights. The carriage was made from two 1/8 thick steel plates through which a 6 bolt is threaded to hold the counterweights. The counterweight consists of twenty steel washers that provide a total counterweight mass of 3.0 kg for both the fixed and swinging configuration. A nail driven into the face of the dowel at one end was attached to a knitted sling that holds the projectile. The treb can be clamped onto a skateboard for trials involving the rolling configuration. Our treb was constructed for under $20 using scrap wood, sheet metal, and commonly available hardware. Construction time was around 3 hours, and required only common hand tools.

Three holes were drilled into the throw arm to provide a choice of axle positions. One of the holes was drilled at a distance of 15 cm from the counterweight end of the throw arm after calculations discussed below showed this location to maximize the angular velocity of the arm at release. Another hole was drilled at the midpoint of the arm, and the third was drilled midway between the other two. Having three options for mounting the axle allowed us to qualitatively verify the results of our analysis which suggested that the release velocity of the projectile is a function of the mass distribution about the axle position.

The treb was painted white so that its motion would be easily resolved in open-shutter strobe photography images. Black dots were painted on the treb at several locations to more easily track the motion of the device in multiple-exposure images of the trebs motion.

III. ANALYSIS

III.1. Fixed-base trebuchet

The fixed counterweight treb is an excellent laboratory device for the introductory curriculum. Application of basic energy conservation principles is straightforward, as is the application of Newtonian dynamics. Watching the trebs motion has sparked many conversations among both students and faculty. In the introductory lab, we have students carry out a simplified analysis of the motion of the fixed-base, fixed counterweight treb shown in 2(a). The throw arm is modeled as a uniform rod of mass m, while the counterweight is treated as a point mass M. Both the throw arm and the counterweight masses are scaled in units of the projectile mass, which is assumed point-like.

Students are asked to compute the moments of inertia of each of the three components of the treb (projectile, counterweight, and throw arm) about the rotation axis which is a distance x from the counterweight. This makes use of the parallel axis theorem, as well as elementary formulae for moments of inertia of rods and point masses. Provided the initial orientation of the throw arm, students can derive an energy balance equation which can then be solved for the angular velocity of the throw arm at any point prior to release of the ball. In our analysis in the introductory course, we neglect the contribution of the sling that holds the projectile, and treat the projectile as a point mass located at the end of the throw arm. Assuming a release angle of $\theta = 90°$, students can show that the angular speed of the throw arm at release is given by

$$\omega_r(x) = \sqrt{\frac{2Mx - (1-x) - m(\frac{1}{2} - x)}{(1-x)^2 + Mx^2 + m(\frac{1}{3} + x^2)}} \qquad (1)$$

Here x is the distance from the counterweight to the axle scaled in units of the length of the throw arm. The angular speed ?r has a maximum for a given combination of m and M at x^*. In general x^* is a complicated function of the masses. However, a useful limit is obtained when the mass of the throw arm can be neglected in comparison to the mass of the counterweight (both scaled in units of the projectile mass).

$$x^* = \frac{1 \pm \sqrt{M}}{1+M},\ m = 0 \qquad (2)$$

While this is not a realistic approximation for full size trebuchets, it is reasonable for some of the heavier projectiles used in our trials and has a simple physical interpretation. Consider the limiting case of $M \to 1$ in which the counterweight mass approaches the projectile mass (and the throw arm has negligible mass). It is clear from Eq. (2) that the optimal angular velocity at release is obtained for $x^* = (0, 1)$, where the axle is located either at the counterweight or at the projectile end of the throw arm. This is a physically reasonable result. For $M > 1$, the positive root in Eq. (2) is taken. Eqns. (1) and (2) can also be obtained by solving $\sum_i \tau_i = I\alpha$ for the throw arm, where the τ_i are the torques acting on the throw arm, I is the moment of inertia of the system, and α is the angular acceleration of the throw arm.

III.2. Sliding-base trebuchet

At this point, we have quantified something that the pre-Newtonian military engineers of the 13th century may have understood quite well: the optimal position of the axle depends on the masses chosen for the counterweights, throw arm, and projectile. It is possible to extend the analysis to considerations of a treb on a sliding or rolling base, and demonstrate that this modification results in increased kinetic energy for the projectile, and a greater effective range. We can understand this by noting that, as the counterweight falls in its arc about the axle, the system mass distribution shifts backward in the opposite direction to the projectile momentum. To conserve a net linear momentum of zero in the x-direction, the base must move in the direction of the projectile, providing additional momentum to the projectile. This will be true whenever the counterweight mass dominates the mass distribution of the trebuchet. By applying both momentum conservation in the x-direction, as well as energy conservation to the trebuchet, students can show that the linear velocity of the projectile at release is increased by as much as 29% over the fixed base treb. Interested readers can find details of the calculations at http://www.carthage.edu/departments/physics/.

III.3. Swinging counterweight trebuchet

What of the swinging counterweight configuration? A detailed analysis of this configuration is left for our upper-division mechanics course, but students at the introductory level can appreciate the essential results. Consider the motions of two otherwise identical trebuchets, one with a fixed counterweight, and one with a swinging counterweight. Both counterweights liberate the same amount of gravitational potential energy. However, the fixed counterweight transfers some of this energy into rotational motion as it is constrained to move about the throw arm. The swinging counterweight falls nearly straight down, without significant rotation, and has very little kinetic energy as the projectile is released. The swinging counterweight transfers more energy into rotational kinetic energy of the throw arm and thus the projectile. Students in our upper-level mechanics course derive the equations of motion for the swinging counterweight treb using Lagrangian methods and demonstrate that the effective range is nearly doubled over the fixed counterweight trebuchet. Again, interested readers are referred to our web page for details of these calculations.

A comparison of the three design configurations is shown in Fig. 3, which was generated with Interactive Physics simulation software.[4]

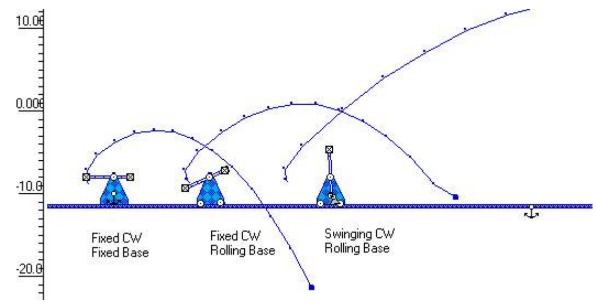

FIG. 3: Computer simulations of three different trebuchet designs clearly show the advantage of the swinging counterweight over the fixed counterweight, and the advantage of the rolling base over the fixed base.

IV. EXPERIMENTS

We carried out a series of experiments, the first of which simply tabulated the effective range of each of the treb configurations. In the range experiments, the mass of the counterweight was held fixed at 3.0 kg, and the projectile was a 100 g steel ball, 3.0 cm in diameter. The release point of the ball was monitored for consistency between trials and between configurations by adjusting the length of the sling. The swinging counterweight configuration is the clear winner in these trials, and putting either treb on a rolling base increases the effective range by 30 %. Medieval engineers likely demonstrated both of these results as early as the late 12th century when drawings of trebuchets began to include hinged counterweights and rolling bases.[3]

In the second set of experiments, the position of the axle along the throw arm was varied to investigate the prediction that ω_r is maximized for an axle position of $x^*(M, m)$. We were able to confirm qualitatively that the range is enhanced by the appropriate choice of axle position.

Finally, we employed stop-action photography of the trebs motion to study the angular velocity of the throw arm for both fixed-counterweight and swinging counterweights, on fixed and rolling bases. A sample photograph is shown in Fig. 4. A flash rate of 60Hz was sufficient to resolve the trebs motion into 23 distinct images. High-speed (1600 ASA) film and a small f/ratio reduced blurring and improved brightness of the images. Video cameras could not be used because the standard 30 Hz frame rate was too slow to capture enough images to use in the analysis. Black dots on the throw arm and base helped locate their centers of mass on the photographs. The position of the projectile prior to release was marked for each image in the multiple exposure prints from which $\omega(t)$ was obtained.

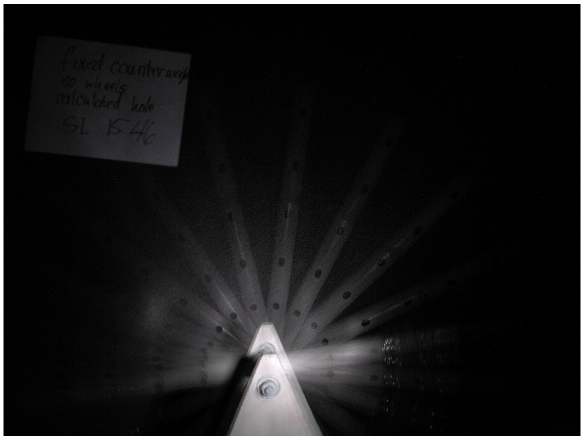

FIG. 4: A strobe light at 2400 fpm resolves the motion of the trebuchet with sufficient detail to allow measurements of the angular velocity of the throw arm.

V. SUMMARY

The mystery surrounding the original designs of trebuchets provides a wonderful historical context in which to study the relatively modern conservation principles. Our students enjoy studying physics in a historical context, especially one in which there is an element of historical sleuthing as they uncover hidden advantages in different design configurations. There is enough physics in the operation of the trebuchet to engage students at both the introductory and advanced level in mechanics courses. Construction of a model trebuchet is inexpensive and straightforward, as is the analysis of its motion. Readers interested in learning more about the history of the trebuchet are referred to the excellent PBS episode of History of Technology[2] and the accompanying Smithsonian article.[3]

VI. ACKNOWLEDGEMENTS

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[1] D. N. Arion, K. M. Crosby, and E. A. Murphy, "Case Study Experiments in the Introductory Physics Curriculum, Phys. Teach. **38**, 373 (2000).

[2] NOVA: "Secrets of Lost Empires, Episode 27, original air date February 1, 2000.

[3] E. Hadingham, "Ready, Aim, Fire!, Smithsonian Magazine, January 2000.

[4] MSC.Working Knowledge, 66 Bovet Road, Suite 200, San Mateo, CA 94402